II. "On the Construction of Life-Tables; illustrated by a New Life-Table of the Healthy Districts of England." By WILLIAM FARR, M.D., F.R.S., Superintendent of the Statistical Department, General Register-Office. Received March 17, 1859.

(Abstract.)

The Transactions of the Royal Society contain the first Life-Table: it was constructed by Halley, who discovered its remarkable properties, and illustrated some of its applications.

With better data improved methods have been found out, and the form has been extended so as to facilitate the solution of various questions.

In deducing the English Life-Tables from the National Returns, I have had occasion to try various methods of construction; and I now propose to describe briefly the nature of the Life-Table, to lay down a simple method of construction, to describe an extension of its form, and to illustrate this by a new Table representing the vitality of the healthiest part of the population of England.

The Life-Table is an instrument of investigation; it may be called a *biometer*, for it gives the exact measure of the duration of life under given circumstances. Such a Table has to be constructed for each district and for each profession, to determine their degrees of salubrity. To multiply these constructions, then, it is necessary to lay down rules, which, while they involve a minimum amount of arithmetical labour, will yield results as correct as can be obtained in the present state of our observations.

A Life-Table represents a generation of men passing through time; and time under this aspect, dating from birth, is called age. In the first column of a Life-Table age is expressed in years, commencing at 0 (birth), and proceeding to 100 or to 110 years, the extreme limit of observed lifetime. Annexed is an outline of the two primary series of the Life-Table, representing the surviving at each year of age (l_x), and the first differences representing the dying (d_x), in annual intervals of age.

Age.	Dying.	Living.	Age.	Dying.	Living.
x.	d_x.	l_x.	x.	d_x.	l_x.
0	10,295	100,000	40	618	63,756
1	3,005	89,705	50	722	57,203
2	1,885	86,700	60	1123	48,855
3	1,305	84,815	70	1825	34,278
4	1,051	83,510	80	1803	14,971
5	847	82,459	90	555	2,265
10	347	79,525	100	19	46
20	552	75,600	106	1	1
30	598	69,792			

The Table may be read thus: of 100,000 children born, 10,295 die in the first year, 89,705 survive.

It will be observed that, upon the hypothesis that the annual births equal the annual deaths in number, and that the law of mortality remains invariable, the series of the living (l_x) can be constructed from the series of numbers (d_x) representing the dying, or from the numbers dying at different ages, as returned in the parish registers. That course was adopted by Halley, and afterwards by Dr. Price, in constructing the Northampton Table. But the hypothesis of an invariable annual number of births equalling the deaths has never been verified by observation, and consequently tables on the plan of Halley's are often exceedingly erroneous. In the healthiest districts of England the births were 29,715, the deaths 17,469 annually: a Table constructed upon that plan, like Dr. Price's, makes the *mean lifetime*—or as it is sometimes called, the expectation of life—for Northampton, 25 years, while the mean lifetime by a correct Northampton Table is 38 years.

It is shown by a diagram that if age (x) is represented by the abscissas, the numbers living (l_x) will be represented by the ordinates of a curve. De Moivre constructed this curve by assuming that the series l_x is from the age 12 to 86, in arithmetical progression; decreasing thus, 74, 73, 72 . . . 3, 2, 1, 0. By another hypothesis, the rate of mortality is assumed to decrease or to increase in geometrical progression at different rates in different periods of life; and it is found that this hypothesis represents the results deduced from the observed facts approximatively.

As v, the velocity, expresses a ratio, so m, the rate of mortality, is the ratio of the number dying to the number living in a unit of

time. Now if y represent the living at a definite age, and r the rate at which the mortality increases at that point of age, then mr^z will be the rate of mortality after the lapse of z units of time. The decrement of y in an infinitely short time will be $dy=ymr^z dz$. This was pointed out by Mr. Gompertz, and Mr. Edmonds subsequently extended the theory. This expression can be integrated, and the final equation of the corrected integral is $y=10^{\frac{k^2m}{\lambda r}(1-r^z)}$; where λ is put for the common logarithm, and k for its modulus.

Either of the hypotheses gives a close approximation to the exact result, within short intervals of time; and the results by the two hypotheses agree at the principal ages after 20, when they can be fairly tested. Thus, if the rate of mortality in any year of age (x to $x+1$) is m, then

$$1+\tfrac{1}{2}m : 1-\tfrac{1}{2}m :: l_x : \frac{1-\frac{1}{2}m}{1+\frac{1}{2}m} \cdot l_x = l_{x+1}.$$

It is here assumed that m is known; and putting p_x for $\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$, we have $p_x l_x = l_{x+1}$; and have thus the means of passing from the numbers living at the age x to the numbers living at the age $x+1$. But upon the other hypothesis, y_0 being $=1$, then

$$p_x = y_1 = 10^{\frac{k^2m}{\lambda r}(1-r)}.$$

Upon the two hypotheses,

$\lambda p_{20} = \bar{1}\cdot9966528$ by the one, and $\bar{1}\cdot9966527$ by the other;

$\lambda p_{40} = \bar{1}\cdot9956263$ by the one hypothesis, and $\bar{1}\cdot9956264$ by the other.

At 80 there is some divergence. I have adopted the latter hypothesis generally; but the other hypothesis is at some of the earlier ages preferred. I have only adopted these hypotheses within the safe limits of a single year in determining eleven values of λp_x, which I have afterwards interpolated by the method of finite differences; thus assuming that the third difference was constant. This gives, I conceive, as near an approximation as we can obtain in the present state of the observations.

λp_x is the first difference of the series λl_x; and consequently it can be constructed by four orders of differences, on the assumption that no error of consequence is caused by assuming that within given limits the fourth difference is constant.

Healthy Districts.

Age.	Persons Dying in each year of age 0—1, 1—2, &c.	Persons Born and Living at each age.	Sum of the Numbers Born and Living at each age (x), from x to the last age in the Table.	Population, or the Living in each year of age 0—1, 1—2, &c.	(1) Sum of the Living in every year, and of the Living of every age (x) and upwards to the last age in the Table, also (2) the Years which the persons (l_x) will live.	(1) The Years which the persons at the age (x) and upwards will live; also (2) the Years which they have lived over x.	Age.
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
0	10,295	100,000	4,951,908	92,611	4,899,665	166,209,701	0
1	3,005	89,705	4,851,908	88,202	4,807,054	161,356,341	1
2	1,885	86,700	4,762,203	85,758	4,718,852	156,593,388	2
3	1,305	84,815	4,675,503	84,162	4,633,094	151,917,415	3
4	1,051	83,510	4,590,688	82,985	4,548,932	147,326,402	4

Mean After-lifetime.

Age.	Persons.				Males.		Females.	
	Carlisle.	England, 1838—44.	63 Healthy Districts.	Sweden.	England.	63 Healthy Districts.	England.	63 Healthy Districts.
0	39	41	49	...	40	49	42	49
20	41	40	43	38	40	43	41	43
40	28	27	30	25	27	29	28	30
60	14	14	15	13	14	15	14	16
80	6	5	6	4	5	5	5	6

The new Life-Tables consist of three sections,—the first representing persons, the second males, and the third females, each section consisting of six columns. On the opposite page is an extract from the first of the sectional Tables.

The properties of these columns are described, and a collection of useful formulas is added. The curious and useful properties of the new column y are illustrated.

The mean annual mortality in England was at the rate of 22 in 1000; but in eighteen districts the mortality ranged from 28 to 36 in 1000, and in sixty-four districts the mortality ranged from 15 to 16 and 17 in 1000; in the other districts it was at intermediate rates. The Table has been constructed from sixty-three of the districts in which the mortality did not exceed 17 in 1000.

Halley first pointed out some of the financial applications of the Life-Table, and the new Table shows that the mean duration of life among large classes of the population exceeds considerably the expectations of life deduced from the ordinary Tables. The science of public health has been developed since Halley's day; and here a new application of the Life-Table is found. If we ascertain at what rate a generation of men die away under the least unfavourable existing circumstances, we obtain *a standard* by which the loss of life under other circumstances is measured; and this I have endeavoured to accomplish in the New Life-Table. And recollecting that the science of public health was almost inaugurated in England by a former President of this Society, who crowned the sanitary discoveries of Captain Cook, I feel assured that the Society will receive with favour this imperfect attempt to supply sanitary inquirers with a scientific instrument.